JOURNAL OF AIRCRAFT
Vol. 31, No. 5, Sept.–Oct. 1994

Two-Point Transonic Airfoil Design Using Optimization for Improved Off-Design Performance

J. O. Hager,* S. Eyi,* and K. D. Lee†
University of Illinois, Urbana, Illinois 61801

A two-point, aerodynamic design method is presented that improves the aerodynamic performance of transonic airfoils over a range of the flight envelope. It couples an Euler flow solver and a numerical optimization tool. The major limitation of single-point design is the poor off-design performance. Two-point design is used to extend the optimized performance range over more of the desired flight envelope. The method is applied to several transonic flow design points, and the results are compared to single-point design results. The secondary design points are chosen by varying the Mach number and the angle of attack. The two-point designs perform better than the single-point design over the design-point range.

Introduction

COMPUTATIONAL fluid dynamics is particularly suited to the design process because many geometries can be evaluated with modest cost in a short period of time. One of the early computational fluid dynamics design applications was based on the cut-and-try approach, where a designer iteratively modifies and verifies a design. Performing such a design method with wind-tunnel models would be costly. And, if the same level of detailed information about the flowfield was required, as can easily be obtained with computational fluid dynamics, the cost would be prohibitive.

Numerical optimization is an improvement over the cut-and-try approach because it is based on a rational, directed, design procedure. Constrained optimization is attractive because a combination of design parameters can be improved, multiple constraints can be imposed, and multipoint design can be performed. It can be used to generate an optimum geometry that has desirable characteristics while satisfying some design constraints.[1,2] One example is to minimize the drag while maintaining the lift and cross-sectional area of the airfoil. One drawback is that numerical optimization may locate a locally optimum solution.

The objective of the present study is to demonstrate a versatile transonic design tool that is capable of two-point design in the Mach-number and angle-of-attack design-point space. Single-point designs have been performed that greatly reduced the drag at the design point, but there was a resulting drag creep (higher drag at slightly lower Mach numbers).[3] By basing the objective on a combination of the drag at two design points, the resulting overall performance of the design can be improved with respect to the single-point design result.[4,5]

This article will present some details of the major technology pieces used in the design, followed by a discussion of the results from several design examples. The design examples will compare the results of the two-point designs with the single-point designs.

Combination of Paper 92-4225 presented at the Design Systems Meeting, Hilton Head, SC, Aug. 24–26, 1992; and Paper 92-4681 presented at the AIAA 4th Symposium on Multidisciplinary Analysis and Optimization, Cleveland, OH, Sept. 21–23, 1992; received Dec. 14, 1992; revision received Feb. 15, 1994; accepted for publication Feb. 17, 1994.

*Research Assistant, Department of Aeronautical and Astronautical Engineering. Student Member AIAA.

†Associate Professor, Department of Aeronautical and Astronautical Engineering. Associate Fellow AIAA.

Design Tools

The flow model used for the present designs is based on the Euler equations. This model is sufficient to include the major features of transonic flow: namely the rotational physics associated with embedded shock waves. The flow code is based on the Jameson finite volume scheme applied to a surface-fitted grid. Centered differences are used for the spatial discretization, and artificial viscosity is added for numerical stability. The time-integration is performed using a fourth-order Runge-Kutta scheme, and local time-stepping is used to accelerate the convergence.

Constrained optimization determines the set of design variables that minimize an objective function subject to specified constraints. The numerical optimization process begins with an initial guess of the design variables, which are then updated through an iterative procedure by determining the best search direction and step size. Gradient information of the objective function and the constraint functions is required during the optimization process. The gradient operator is the sensitivity of the function to changes in the design variables, and it is calculated using finite differences. When no constraints are active, the search directions are determined using the Fletcher-Reeves conjugate direction method. The present design uses a commercially available constrained optimization tool.[6]

Design Variables

The design process begins with an initial airfoil. The airfoil geometry is then modified by adding smooth perturbations.

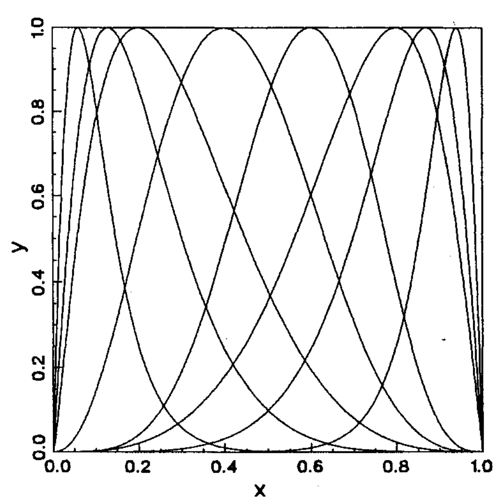

Fig. 1 Sinusoidal base functions used to perturb the geometry.

Table 1 RAE 2822, Mach-design results (M_1 = 0.726, α_1 = 2.44 deg), (M_{2A} = 0.720, α_{2A} = 2.44 deg), (M_{2B} = 0.700, α_{2B} = 2.44 deg)

		Initial	Final	Δ, %
	1) Single-point design at 1, w = 1.00[a]			
1	C_l	0.8924	0.8947	0.26
	C_d	0.0112	0.0054	−51.79
	C_m	−0.1205	−0.1157	−3.95
2A	C_l	0.8776	0.8611	−1.88
	C_d	0.0083	0.0062	−24.73
	C_m	−0.1136	−0.1133	−0.26
2B	C_l	0.8223	0.7961	−3.19
	C_d	0.0039	0.0062	60.98
	C_m	−0.1025	−0.1095	6.80
	A	0.0780	0.0787	0.93
	2) Two-point design at 1 and 2A, w = 0.50[b]			
1	C_l	0.8924	0.8933	0.10
	C_d	0.0112	0.0058	−47.88
	C_m	−0.1205	−0.1158	−3.85
2A	C_l	0.8776	0.8676	−1.15
	C_d	0.0083	0.0049	−41.20
	C_m	−0.1136	−0.1106	−2.65
2B	C_l	0.8223	0.8002	−2.70
	C_d	0.0039	0.0052	35.35
	C_m	−0.1025	−0.1093	6.65
	A	0.0780	0.0790	1.34
	3) Two-point design at 1 and 2B, w = 0.50[c]			
1	C_l	0.8924	0.8934	0.11
	C_d	0.0112	0.0056	−50.03
	C_m	−0.1205	−0.1170	−2.90
2A	C_l	0.8776	0.8645	−1.50
	C_d	0.0083	0.0050	−39.57
	C_m	−0.1136	−0.1123	−1.13
2B	C_l	0.8223	0.8004	−2.67
	C_d	0.0039	0.0052	34.98
	C_m	−0.1025	−0.1093	6.63
	A	0.0780	0.0780	0.00

[a]N_{cyc} = 3, N_{geom} = 63. [b]N_{cyc} = 3, N_{geom} = 61. [c]N_{cyc} = 6, N_{geom} = 105.

The geometry perturbation is defined by a linear combination of the base functions f_k as

$$\Delta y(x) = \sum_{k=1}^{K} \delta_k f_k(x) \tag{1}$$

where the weighting coefficients δ_k are the K design variables to optimize. In the present study, 16 total base functions were used, 8 on both the upper and lower sides of the airfoil. The base functions are composed of the following sinusoidal functions:

$$\begin{aligned} f_k(x) &= \sin[\pi(1-x)^{e(k)}], \quad k = 1, 2 \\ f_k(x) &= \sin^3[\pi x^{e(k)}], \quad k = 3, \ldots, 6 \\ f_k(x) &= \sin[\pi x^{e(k)}], \quad k = 7, 8 \\ e(k) &= \frac{\ln(0.5)}{\ln(1-x_k)}, \quad k = 1, 2 \\ e(k) &= \frac{\ln(0.5)}{\ln(x_k)}, \quad k = 3, \ldots, 8 \end{aligned} \tag{2}$$

where x_k is the location of the maximum height of the base function. Figure 1 shows the base functions used with x_k = 0.006, 0.13, 0.2, 0.4, 0.6, 0.8, 0.87, 0.94.

The performance of the design process depends on the number and shape of the base functions. Increasing the number of base functions may improve the quality of the design, but it will increase the cost of the design. Using base functions that are not smooth can cause geometry changes that are not smooth, and this will adversely effect the design result.

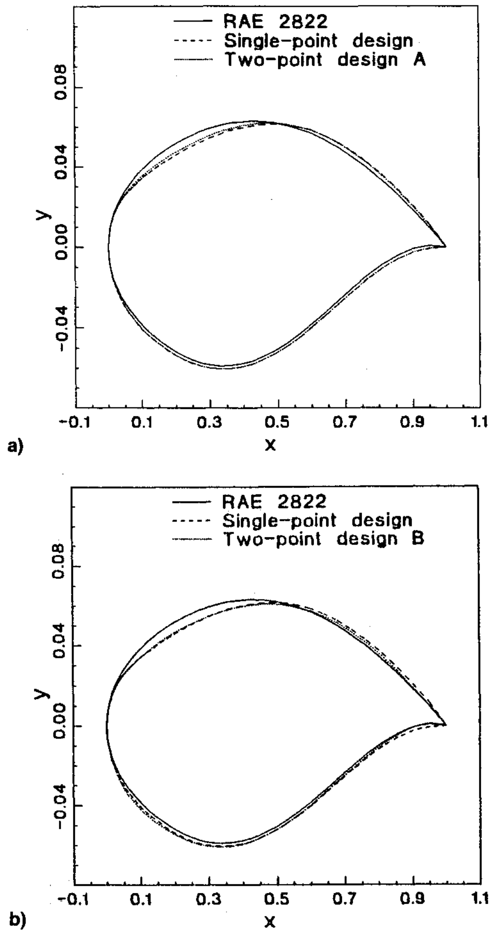

Fig. 2 RAE 2822, Mach design: airfoil geometry comparisons (M_1 = 0.726, α_1 = 2.44 deg), (M_{2A} = 0.720, α_{2A} = 2.44 deg), (M_{2B} = 0.700, α_{2B} = 2.44 deg): a) design A and b) design B results.

Results and Discussion

The objective of the present design is to obtain an airfoil geometry that produces minimum transonic drag at two flight conditions: 1) the primary design point, and 2) the secondary design point. This is to be achieved subject to inequality constraints on the lift at the primary design point and the airfoil area, so that these values are not decreased by the optimized airfoil. To put this in the mathematical form

Minimize

$$F(\bar{\delta}) = wC_{d_1} + (1-w)C_{d_2}, \qquad 0 \le w \le 1$$

Subject to

$$\begin{aligned} g_1(\bar{\delta}) &= 1 - (C_{l_1}/C_{l_{1_0}}) \le 0 \\ g_2(\bar{\delta}) &= 1 - (A/A_0) \le 0 \end{aligned} \tag{3}$$

where C_d and C_l are the drag and lift coefficients, respectively, A is the airfoil area, and the subscript 0 indicates the initial value. The weighting parameter w controls the influence that the two design points will have on the design.

All designs were performed on the RAE 2822 airfoil using a 128 × 32 grid with 80 points on the surface. The primary design-point flow condition was selected to be case 6 of the AGARD experimental data base (M_1 = 0.726, α_1 = 2.44

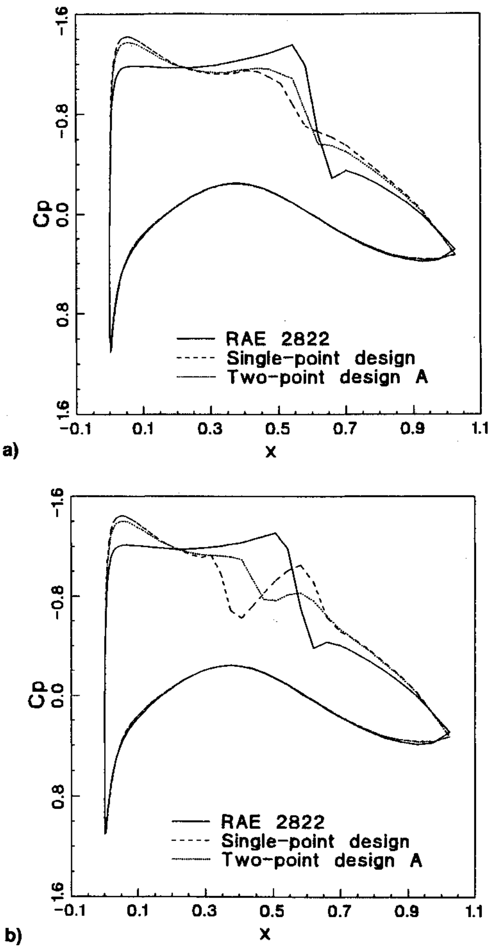

Fig. 3 RAE 2822, Mach design results for two-point design A (M_1 = 0.726, α_1 = 2.44 deg), (M_{2A} = 0.720, α_{2A} = 2.44 deg). Surface pressure comparison at condition a) 1 and b) 2A.

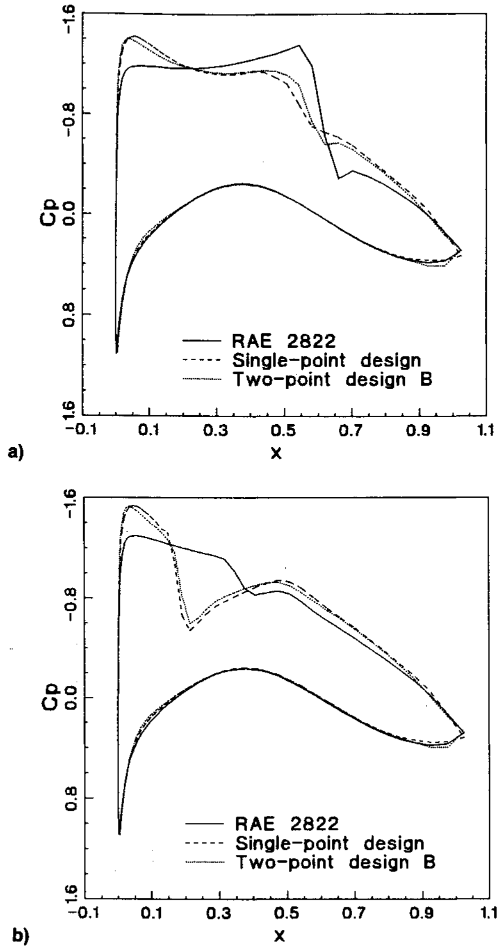

Fig. 4 RAE 2822, Mach design results for two-point design B (M_1 = 0.726, α_1 = 2.44 deg), (M_{2B} = 0.700, α_{2B} = 2.44 deg). Surface pressure comparison at condition a) 1 and b) 2B.

deg). The flow calculations were performed until the average residual was reduced to a specified value. The values N_{cyc} and N_{geom} represent the number of design cycles and the number of geometries evaluated, respectively, and provide a measure of the design cost.

The first design case was performed using a Mach number variation for the secondary design point. Two secondary design points were used: A, M_{2A} = 0.720, α_{2A} = 2.44 deg; B, M_{2B} = 0.700, α_{2B} = 2.44 deg. Table 1 shows the changes in the design parameters for the three cases. The single-point design produces better primary design-point performance than either of the two-point designs. However, both two-point design cases greatly reduce the drag at both secondary design points, with respect to the single-point design, while maintaining a smaller drag at the primary design-point. Figures 3 and 2a show the pressure distributions and geometries for the case A secondary design point. Here, the usefulness of multipoint design is evident in the pressure distribution at the secondary design point. The single-point design has two fairly strong shocks, whereas the two-point design has one fairly strong and one very weak shock. Figures 4 and 2b show the pressure distributions and geometries for the case B secondary design point. Here, the advantage of multipoint design is not as clear. At the primary design point, the two-point design has a slightly stronger shock than the single-point design, and at the secondary design point, the two-point design has a slightly weaker shock than the single-point design. Figure 5, however, does show the advantage of multipoint design by presenting the off-design performance. The single-point design has a large drag creep that begins immediately away from the primary design point. The two-point design A also has a drag creep, but it does not begin until before the secondary

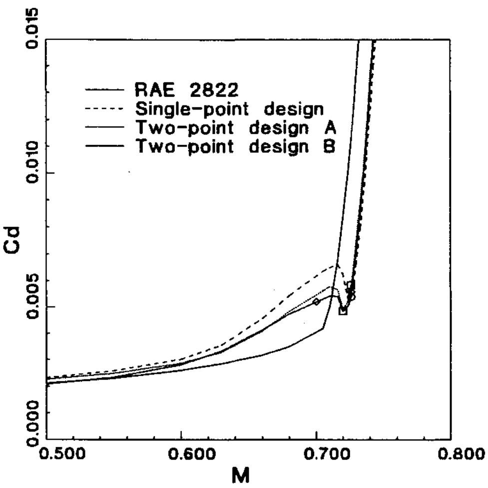

Fig. 5 RAE 2822, Mach design: drag evaluation at off-design conditions (M_1 = 0.726, α_1 = 2.44 deg), (M_{2A} = 0.720, α_{2A} = 2.44 deg), (M_{2B} = 0.700, α_{2B} = 2.44 deg). (Markers designate design points.)

design-point A. The two-point design B is very similar to the two-point design A in the drag creep region, but the maximum drag in the drag creep area has been reduced.

The second design case was performed using an angle-of-attack variation for the secondary design point. The secondary design point was chosen to be at the condition where the drag of the single-point design becomes greater than the original airfoil (M_2 = 0.726, α_2 = 2.00 deg). Several designs were performed to evaluate the effect of the weighting parameter w. Table 2 shows the changes in the design parameters for

Table 2 RAE 2822, Mach-design results (M_1 = 0.726, α_1 = 2.44 deg), (M_2 = 0.726, α_2 = 2.44 deg)

		Initial	Final	Δ, %
	1) Single-point design at 1, w = 1.00[a]			
1	C_l	0.8924	0.8947	0.26
	C_d	0.0112	0.0054	−51.79
	C_m	−0.1205	−0.1157	−3.95
2	C_l	0.7974	0.7733	−3.03
	C_d	0.0063	0.0066	4.50
	C_m	−0.1169	−0.1234	5.51
	A	0.0780	0.0787	0.93
	2) Two-point design at 1 and 2, w = 0.70[b]			
1	C_l	0.8924	0.8926	0.03
	C_d	0.0112	0.0051	−54.13
	C_m	−0.1205	−0.1167	−3.16
2	C_l	0.7974	0.7741	−2.93
	C_d	0.0063	0.0060	−5.33
	C_m	−0.1169	−0.1241	6.19
	A	0.0780	0.0792	1.59
	3) Two-point design at 1 and 2, w = 0.50[c]			
1	C_l	0.8924	0.8960	0.41
	C_d	0.0112	0.0075	−33.47
	C_m	−0.1205	−0.1136	−5.71
2	C_l	0.7974	0.7864	−1.38
	C_d	0.0063	0.0045	−28.19
	C_m	−0.1169	−0.1124	−3.86
	A	0.0780	0.0796	2.00

[a] N_{cyc} = 3, N_{geom} = 63. [b] N_{cyc} = 5, N_{geom} = 102. [c] N_{cyc} = 3, N_{geom} = 55.

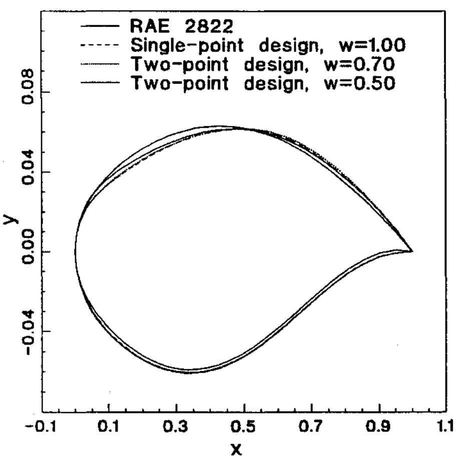

Fig. 6 RAE 2822, alpha-design: airfoil geometry comparison (M_1 = 0.726, α_1 = 2.44 deg), (M_2 = 0.726, α_2 = 2.44 deg).

the single-point design and two two-point design cases: w = 1.00, 0.70, 0.50. This is an example of the single-point design locating a locally optimum solution: the two-point design with w = 0.70 produces the least drag at the primary design point using the same constraints. The two-point design cases, however, also reduce the drag at the secondary design point while maintaining a smaller drag at the primary design point. The cost of the two-point designs varies significantly with the weighting parameter. The even-weighted design case, w = 0.50, costs less to perform than a single-point design, in terms of the number of geometries evaluated, but it required about 50% more computer time because two flow solutions must be obtained for each geometry. The w = 0.70 case required 250% more computer time than the w = 1.00 case. The geometries, shown in Fig. 6, indicate that the single- and two-point designs have similar geometries, but the w = 0.50 design is thicker near the leading edge. Figure 7 shows the pressure distributions at the two-design points. At the primary design

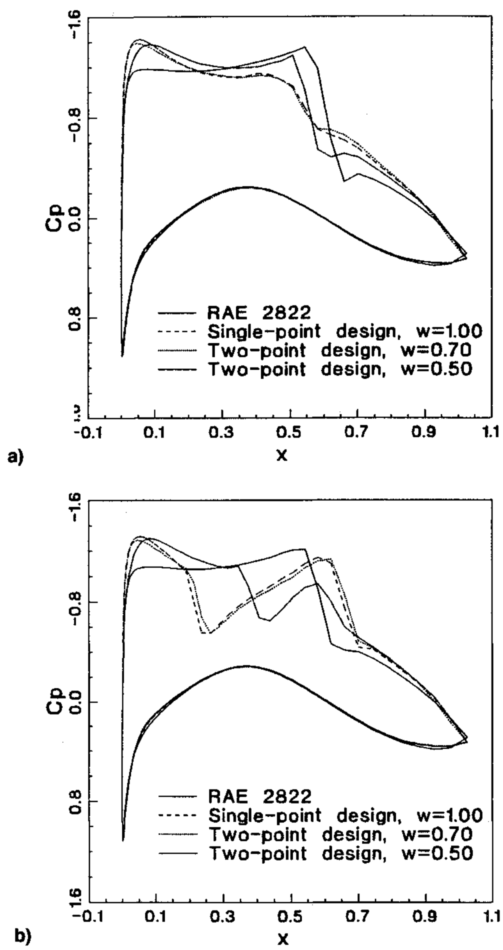

Fig. 7 RAE 2822, alpha-design results (M_1 = 0.726, α_1 = 2.44 deg), (M_2 = 0.726, α_2 = 2.00 deg). Surface pressure comparison at condition a) 1 and b) 2.

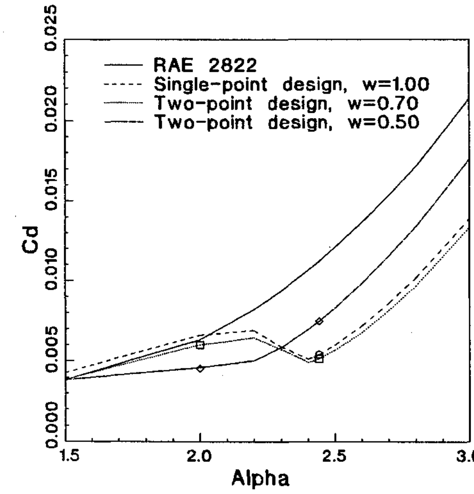

Fig. 8 RAE 2822, alpha-design: drag evaluation at off-design conditions (M_1 = 0.726, α_1 = 2.44 deg), (M_2 = 0.726, α_2 = 2.00 deg). (Markers designate design points.)

point, the single-point design and the w = 0.70 design produce the best pressure distribution, because the shocks are the weakest, and the w = 0.50 design produces a smaller improvement over the original airfoil. However, at the secondary design point, the single-point design and the w = 0.70 design produce two fairly strong shocks (the w = 0.70 design shocks are further aft, which produces less drag due to the more favorable geometry slope), and the w = 0.50 design produces two weaker shocks. The main advantage of two-point design can be seen in Fig. 8. The single-point design

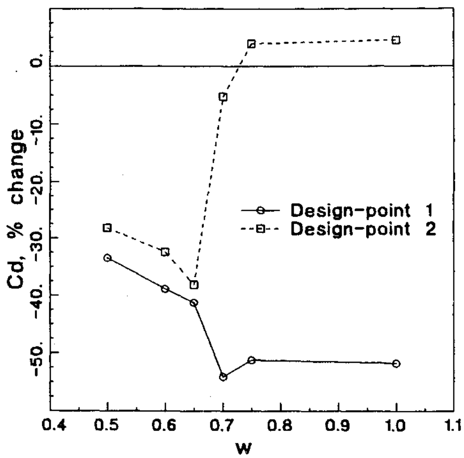

Fig. 9 RAE 2822, alpha-design: drag reduction at the two design points as a function of *w*.

has a large drag creep that begins immediately away from the primary design point. The $w = 0.50$ two-point design removes this drag creep, but at the expense of a reduced drag reduction at the primary design point. The $w = 0.70$ two-point design represents a compromise: 1) large reduced drag at the primary design point and 2) a small reduced drag at the secondary design point. Figure 9 shows the drag reduction at the two design points as a function of the weighting parameter w, and demonstrates the nonlinear effect of the weighting parameter. The $w = 0.65$ design produces a more-optimum, even-weighted design than the actual $w = 0.50$ design. The $w = 0.75$ design is almost identical to the $w = 1.00$, or single-point, design. Between $w = 0.65$ and $w = 0.70$, the character of the solution changes from being like a $w = 0.50$ solution to a $w = 1.00$ solution.

From these two cases, it can be seen that a two-point design produces better performance than a single-point design over the design-point range. However, because the RAE 2822 airfoil is already optimized for transonic flow, even the two-point designs presented do not universally improve the performance of the airfoil. Also, the effect of the weighting parameter is highly nonlinear, which makes the resulting design performance difficult to predict. Similar results were obtained when the two-point design method was applied to other airfoils and flow conditions.

Conclusions

The two-point design method presented is an effective and cost-efficient tool for designing transonic airfoils to optimize performance over a range of flight conditions. The major limitation of single-point design is the poor off-design performance. Using two design points improves the performance of the design with respect to a single-point design for the desired flight-condition range. Using two design points does not necessarily increase the cost of the design in terms of the number of geometries evaluated. However, two flow solutions must be obtained for each geometry that is evaluated. Using a flow model based on the Euler equations captures the major transonic flow features while retaining relatively low cost. However, a more complete flow model, the Navier-Stokes equations, should be used to improve the design reliability. Even though the two-point designs reported here are better than the single-point designs, a better selection of objective functions and constraints could be made to improve the designs even further. The method can be applied to any airfoil at any set of design points.

References

[1]Hicks, R. M., Vanderplaats, G. N., Murman, E. M., and King, R. R., "Airfoil Section Drag Reduction at Transonic Speeds by Numerical Optimization," NASA TM X-73,097, 1976.

[2]Chen, M. S., and Chow, C. Y., "Numerical Optimization Design of Transonic Airfoils," *Numerical Methods in Laminar and Turbulent Flow*, Vol. 6, Pt. 1, Pineridge Press International, Swansea, Wales, UK, 1989, pp. 905–915.

[3]Lee, K. D., and Eyi, S., "Transonic Airfoil Design by Constrained Optimization," *Proceedings of the AIAA 9th Applied Aerodynamics Conference* (Baltimore, MD), Vol. 2, AIAA, Washington, DC, 1991, pp. 677–687.

[4]Hager, J. O., Eyi, S., and Lee, K. D., "Multi-Point Design of Transonic Airfoils Using Optimization," AIAA Paper 92-4225, Aug. 1992.

[5]Hager, J. O., Eyi, S., and Lee, K. D., "A Multi-Point Optimization for Transonic Airfoil Design," *Proceedings of the AIAA 4th Symposium on Multidisciplinary Analysis and Optimization* (Cleveland, OH), AIAA, Washington, DC, 1992, pp. 13–20.

[6]Vanderplaats, G. N., and Hansen, S. R., *DOT Users Manual*, VMA Engineering, Goleta, CA, 1989.